

GCE

Edexcel GCE

Core Mathematics C3 (6665)

Summer 2005

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Mark Scheme (Results)

June 2005
6665 Core C3
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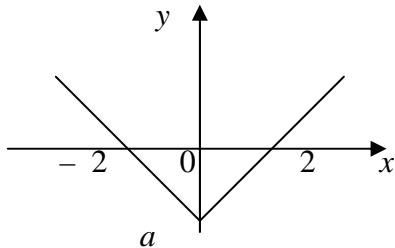
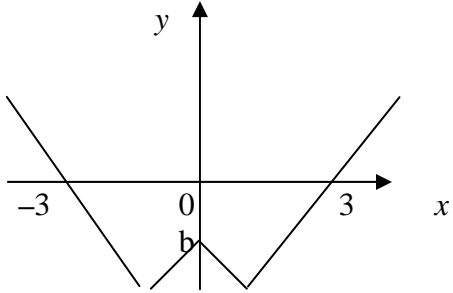
Question Number	Scheme	Marks
1. (a)	Dividing by $\cos^2 \theta$: $\frac{\sin^2 \theta}{\cos^2 \theta} + \frac{\cos^2 \theta}{\cos^2 \theta} \equiv \frac{1}{\cos^2 \theta}$ Completion : $1 + \tan^2 \theta \equiv \sec^2 \theta$ (no errors seen)	M1 A1 (2)
(b)	Use of $1 + \tan^2 \theta = \sec^2 \theta$: $2(\sec^2 \theta - 1) + \sec \theta = 1$ $[2\sec^2 \theta + \sec \theta - 3 = 0]$ Factorising or solving: $(2\sec \theta + 3)(\sec \theta - 1) = 0$ $[\sec \theta = -\frac{3}{2} \text{ or } \sec \theta = 1]$ $\theta = 0$ $\cos \theta = -\frac{2}{3}$; $\theta_1 = 131.8^\circ$ $\theta_2 = 228.2^\circ$ $[A1ft \text{ for } \theta_2 = 360^\circ - \theta_1]$	M1 M1 B1 M1 A1 A1√ (6) [8]

Question Number	Scheme	Marks
<p>2. (a)</p> <p>(i) $6 \sin x \cos x + 2 \sec 2x \tan 2x$ or $3 \sin 2x + 2 \sec 2x \tan 2x$</p> <p>(ii) $3(x + \ln 2x)^2 \left(1 + \frac{1}{x}\right)$</p> <p>(b)</p>	<p>[M1 for $6 \sin x$]</p> <p>[B1 for $3(x + \ln 2x)^2$]</p> <p>Differentiating numerator to obtain $10x - 10$ Differntiating denominator to obtain $2(x-1)$</p> <p>Using quotient rule formula correctly:</p> <p>To obtain $\frac{dy}{dx} = \frac{(x-1)^2(10x-10) - (5x^2-10x+9)2(x-1)}{(x-1)^4}$</p> <p>Simplifying to form $\frac{2(x-1)[5(x-1)^2 - (5x^2-10x+9)]}{(x-1)^4}$</p> <p>$= -\frac{8}{(x-1)^3}$ * (c.s.o.)</p> <p>Alternatives for (b) Either Using product rule formula correctly: Obtaining $10x - 10$ Obtaining $-2(x-1)^{-3}$ To obtain $\frac{dy}{dx} = (5x^2-10x+9)\{-2(x-1)^{-3}\} + (10x-10)(x-1)^{-2}$</p> <p>Simplifying to form $\frac{10(x-1)^2 - 2(5x^2-10x+9)}{(x-1)^3}$</p> <p>$= -\frac{8}{(x-1)^3}$ * (c.s.o.)</p> <p>Or Splitting fraction to give $5 + \frac{4}{(x-1)^2}$ Then differentiating to give answer</p>	<p>M1A1A1 (3)</p> <p>B1M1A1 (3)</p> <p>B1 B1</p> <p>M1 A1</p> <p>M1 A1 (6)</p> <p>[12]</p> <p>M1 B1 B1 A1 cao</p> <p>M1 A1 (6)</p> <p>M1 B1 B1 M1 A1 A1 (6)</p>

Question Number	Scheme	Marks
3(a)	$\frac{5x + 1}{(x + 2)(x - 1)} - \frac{3}{x + 2}$ $= \frac{5x + 1 - 3(x - 1)}{(x + 2)(x - 1)}$ <p>M1 for combining fractions even if the denominator is not lowest common</p> $= \frac{2x + 4}{(x + 2)(x - 1)} = \frac{2(x + 2)}{(x + 2)(x - 1)} = \frac{2}{x - 1} \quad *$ <p>M1 must have linear numerator</p>	<p>B1</p> <p>M1</p> <p>M1 A1 cso</p> <p>(4)</p>
(b)	$y = \frac{2}{x - 1} \Rightarrow xy - y = 2 \Rightarrow xy = 2 + y$ $f^{-1}(x) = \frac{2 + x}{x} \quad \text{o.e.}$	<p>M1A1</p> <p>A1</p> <p>(3)</p>
(c)	$fg(x) = \frac{2}{x^2 + 4} \quad (\text{attempt}) \quad \left[\frac{2}{"g" - 1} \right]$ <p>Setting $\frac{2}{x^2 + 4} = \frac{1}{4}$ and finding $x^2 = \dots; \quad x = \pm 2$</p>	<p>M1</p> <p>M1; A1</p> <p>(3)</p> <p>[10]</p>

Question Number	Scheme	Marks
4	<p>(a) $f'(x) = 3e^x - \frac{1}{2x}$</p> <p>(b) $3e^x - \frac{1}{2x} = 0$</p> $\Rightarrow 6\alpha e^\alpha = 1 \quad \Rightarrow \alpha = \frac{1}{6} e^{-\alpha} \quad (*)$ <p>(c) $x_1 = 0.0613\dots, x_2 = 0.1568\dots, x_3 = 0.1425\dots, x_4 = 0.1445\dots$</p> <p>[M1 at least x_1 correct, A1 all correct to 4 d.p.]</p> <p>(d) Using $f'(x) = 3e^x - \frac{1}{2x}$ with suitable interval</p> <p>e.g. $f'(0.14425) = -0.0007$ $f'(0.14435) = +0.002(1)$</p> <p>Accuracy (change of sign and correct values)</p>	<p>M1A1A1 (3)</p> <p>M1</p> <p>A1 cso (2)</p> <p>M1 A1 (2)</p> <p>M1</p> <p>A1 (2)</p> <p>[9]</p>

Question Number	Scheme	Marks
5. (a)	$\cos 2A = \cos^2 A - \sin^2 A \quad (+ \text{ use of } \cos^2 A + \sin^2 A \equiv 1)$ $= (1 - \sin^2 A) - \sin^2 A = 1 - 2 \sin^2 A \quad (*)$	M1 A1 (2)
(b)	$2 \sin 2\theta - 3 \cos 2\theta - 3 \sin \theta + 3 \equiv 4 \sin \theta \cos \theta - 3(1 - 2 \sin^2 \theta) - 3 \sin \theta + 3$ $\equiv 4 \sin \theta \cos \theta + 6 \sin^2 \theta - 3 \sin \theta$ $\equiv \sin \theta (4 \cos \theta + 6 \sin \theta - 3) \quad (*)$	B1; M1 M1 A1 (4)
(c)	$4 \cos \theta + 6 \sin \theta \equiv R \sin \theta \cos \alpha + R \cos \theta \sin \alpha$ <p>Complete method for R (may be implied by correct answer)</p> $[R^2 = 4^2 + 6^2, R \sin \alpha = 4, R \cos \alpha = 6]$ $R = \sqrt{52} \text{ or } 7.21$ <p>Complete method for α; $\alpha = 0.588$ (allow 33.7°)</p>	M1 A1 M1 A1 (4)
(d)	$\sin \theta (4 \cos \theta + 6 \sin \theta - 3) = 0$ $\theta = 0$ $\sin(\theta + 0.588) = \frac{3}{\sqrt{52}} = 0.4160.. \quad (24.6^\circ)$ $\theta + 0.588 = (0.4291), 2.7125 \text{ [or } \theta + 33.7^\circ = (24.6^\circ), 155.4^\circ]$ $\theta = 2.12 \text{ rad}$	M1 B1 M1 dM1 A1 (5) [15]

Question Number	Scheme	Marks
6. (a)	 <p style="text-align: center;"> y x -2 0 2 a </p>	<p>Translation ← by 1</p> <p>Intercepts correct</p> <p>M1</p> <p>A1 (2)</p>
(b)	 <p style="text-align: center;"> y x -3 0 3 b </p>	<p>$x \geq 0$, correct “shape” provided graph is not original graph</p> <p>Reflection in y-axis</p> <p>Intercepts correct</p> <p>B1</p> <p>B1√</p> <p>B1 (3)</p>
(c)	<p>$a = -2, b = -1$</p>	<p>B1B1 (2)</p>
(d)	<p>Intersection of $y = 5x$ with $y = -x - 1$</p> <p style="text-align: center;">Solving to give $x = -\frac{1}{6}$</p> <p>[Notes: (i) If both values found for $5x = -x - 1$ and $5x = x - 3$, or solved algebraically, can score 3 out of 4 for $x = -\frac{1}{6}$ and $x = -\frac{3}{4}$; required to eliminate $x = -\frac{3}{4}$ for final mark. (ii) Squaring approach: M1 correct method, $24x^2 + 22x + 3 = 0$ (correct 3 term quadratic, any form) A1 Solving M1, Final correct answer A1.]</p>	<p>M1A1</p> <p>M1A1 (4)</p> <p style="text-align: right;">[11]</p>

7. (a)	<p>Setting $p = 300$ at $t = 0 \Rightarrow 300 = \frac{2800a}{1+a}$</p> <p>$(300 = 2500a)$; $a = 0.12$ (c.s.o) *</p>	M1 dM1A1 (3)
(b)	$1850 = \frac{2800(0.12)e^{0.2t}}{1 + 0.12e^{0.2t}} ; \quad e^{0.2t} = 16.2\dots$	M1A1
	<p>Correctly taking logs to $0.2 t = \ln k$ $t = 14$ (13.9..)</p>	M1 A1 (4)
(c)	<p>Correct derivation: (Showing division of num. and den. by $e^{0.2t}$; using a)</p>	B1 (1)
(d)	<p>Using $t \rightarrow \infty$, $e^{-0.2t} \rightarrow 0$,</p> $p \rightarrow \frac{336}{0.12} = 2800$	M1 A1 (2)
		[10]